**PF ASSIGNMENT# 1**

**BY ROLL N0 24018 & 24030**

QUESTION NO 1.1

(BY ROLL NO 24018)

| KEY TABLE | |
| --- | --- |
| 8 LITER DRUM | A |
| 5 LITER DRUM | B |

For 3 Liter Oil   
(Self written algorithm):

* Fill A with oil till the top.
* Now transfer oil from A to B until B is completely full.
* The remaining oil in A is **3 liters.**

(CHAT GPT PSEUDOCODE):

* Fill container A with oil until it's full.
* Measure the total amount of oil in A.
* Transfer oil from A to B until B is completely full.
* Measure the remaining oil in A.
* Remaining oil in A is **3 liters.**

**For 2 Liter Oil**(Self written algorithm):

* Fill B to the brim with oil.
* Pour oil from B into A.
* Refill B with oil.
* Transfer oil from B into A until A is completely full.
* Remaining oil left in B is **2 liters.**

(CHATGPT PSEUDOCODE):

* Fill container B to the brim with oil.
* Transfer oil from B into A.
* Refill B with oil.
* Transfer oil from B into A until A is completely full.
* Measure the remaining oil in B.
* Remaining oil in B is **2 liters.**

**For 1 Liter Oil**   
(Self written algorithm):

* Fill A with oil till the top.
* Now transfer oil from A to B until B is completely full.
* The remaining oil in A is 3 liters.
* Empty B.
* Pour oil from A to B until A is empty. (B has 3 liters now)
* Refill A completely.
* Transfer oil from A to B again, until B is completely full. (A has 6 liters remaining now
* Empty B again.
* Pour oil from A to B again until filled to the top.
* Remaining oil in A is **1 liter.**

(CHATGPT PSEUDOCODE):

* Fill A to the top.
* Transfer oil from A to B until B is full.
* Empty B.
* Transfer remaining oil from A to B (B has 3 liters now).
* Refill A to the top.
* Transfer oil from A to B until B is full (A has 6 liters remaining now).
* Empty B.
* Transfer oil from A to B until B is full (**A has 1 liter remaining).**  
  **For 4 Liter Oil**(Self written algorithm) :
* Fill B to the brim with oil.
* Pour oil from B into A.
* Refill B with oil.
* Transfer oil from B into A until A is completely full.
* Remaining oil left in B is 2 liters.
* Repeat this procedure to obtain **4 liters** in total.   
    
  (Chatgpt pseudocode):
* Fill B to the brim with oil.
* Transfer oil from B to A until A is full.
* Refill B to the brim with oil.
* Transfer oil from B to A until A is full.
* Remaining oil in B = 2 liters
* Repeat this procedure to obtain **4 liters** of oil in total.

For 6 Liter Oil :

(Self written algorithm):

* Fill A with oil till the top.
* Now transfer oil from A to B until B is completely full.
* The remaining oil in A is 3 liters.
* Repeat this process until you have a total of **6 liters.**

(Chatgpt pseudocode)

* Fill A to the top.
* Transfer oil from A to B until B is full
* Remaining oil in A = 3 liters
* Repeat this process to accumulate a total of **6 liters.**

FOR 7 liter oil:  
 (SELF WRITTEN ALGORITHM)

* Fill B to the brim with oil.
* Pour oil from B into A.
* Refill B with oil.
* Transfer oil from B into A until A is completely full.
* Remaining oil left in B is 2 liters.
* Transfer it to A.
* Refill B till the top.
* Transfer oil from B to A again.
* Total oil in A now is **7 liters.**

(PSEUDOCODE BY CHATGPT):

* Fill B to maximum capacity.
* Transfer oil from B to A.
* Refill B to maximum capacity.
* Transfer oil from B to A until A is full.
* Remaining oil in B = 2 LITERS
* Transfer remaining oil in B to A.
* Refill B to maximum capacity.
* Transfer oil from B to A.
* Total oil in A = **7 LITERS**

QUESTION NO : 1.2

(BY ROLL N0 24030)

CASE : 1 FOR 20 STUDENTS

SOLUTION :

There are 20 students in class , each student shakes hands with every other student exactly once

* Student 1 shake hands with 19 other students
* Student 2 shake hands with 18 other students , excluding student 1
* Student 3 shake hands with 17 other students , excluding student 1 and 2
* Student 4 shake hands with 16 other students , and so on

To find the total number of handshakes for 20 students we will use the formula of sum of first **N** positive integers :

So for 20 students = 20 ( 20 - 1) / 2

= **190 handshakes**

CASE : 2 FOR N STUDENTS

SOLUTION :

If there are N students in class , each student will handshake with N-1 students

* By using the above formula we can find find total number of handshakes for N students
* Total handshakes =

ARGUMENT:

We are using this particular formula because :

* You are counting every possible handshake once.
* The factor **½** is there because each handshake involves two people, so without it, you'd be counting each handshake twice.

CHAT GPT VERSION ( WITH CHANGINGS)

CASE : 1 PSEUDO CODE FOR 20 STUDENTS

SOLUTION :

1. Input X = 20 // X is number of students
2. Calculate total handshakes by using the formula = N\* ( N - 1) / 2
3. Put X = 20 in place of N
4. Output = total no of handshakes for 20 students
5. Result = 190 handshakes

CASE : 2 PSEUDO CODE FOR N STUDENTS

SOLUTION :

1. Input N // N is number of students
2. Find total handshakes by using the formula = N\* ( N - 1) / 2
3. Output = total no of handshakes for N students

**QUESTION NO: 1.3**

**(BY ROLL NO 24018)**

SOLUTION:

* We have a **8x8** chessboard which means a total of **64** 1x1 boxes
* To find 2 by 2 squares, we will be multiplying 7x7 (Because a 2x2 square fits into a 7x7 grid of starting points).
* For 3x3 squares, we will multiply 6x6.
* This creates a pattern in the following manner :  
   **(8x8)+(8-1 x 8-1)+(8-2 x 8-2)+....**
* The general formula for the number of k x k squares on an N x N chessboard is   
  **(N - k + 1) x (N - k + 1)**
* The final answer will be **204.**

**PSEUDOCODE BY CHATGPT**

* N = Size of the chessboard
* Total squares = 0
* For k from 1 to N:   
   a. Number of k by k squares = (N - k + 1) \* (N - k + 1)   
   b. Number of k by k squares to total squares
* We will get a total no of squares.

QUESTION NO : 1.4

(BY ROLL NO 24030)

SOLUTION:

* We want to minimize the number of drops needed to find the highest floor out of **100** from which the ball can be dropped without breaking.
* If we drop the first ball from a lower floor then **50th** e.g **15th floor** and then skip a few floors e.g increase by 14 or 15 and so on, so that if the first ball breaks at any of the floors then there are a minimum number of floors needed to be checked with the second ball.
* So, if we assume the first floor from which we drop the ball to be **"x ",** then we need to find the smallest value of **x** such that if the first ball breaks at **xth** floor then the number of the drops needed for second ball from **1 to "x-1"** floors below that **xth** floor are minimized. If it doesn't break we will move up by **x+(x-1)th** floor and if it breaks we will check **x-2** floors with a second ball and so on.
* We know that the total no of drops from both balls should not exceed 100. So, if we start dropping a ball from the xth floor and if it doesn't break then we go up by decreasing the number of floors skipped each time i.e, x, x-1, x-2 and so on. Means the floors to be checked if the ball doesn't break will be **x , x + (x-1 ), x +( x-1 )+ (x-2)** and so on .
* To do this, sum of drops should give equation like this:

* And this is equal to **x(x+1)/2 >= 100**
* Solving this inequality :

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* So the highest floor from which the first ball will be dropped is **14** and as discussed above if it doesn't break then we will go to the next floor : **14 + (14-1) = 27** and then **14+13+12 =39** and so on till the ball breaks. (The floors tested by following this will be **14, 27, 39 , 50 60 69 77 84 90 95 99 100**) Then use the second ball to test floors from the last **xth** floor from which you dropped the ball without breaking till the floors it breaks
* First calculate the drops for worst case scenario:

Worst case scenario will be if the ball breaks at **14th floor** on the first drop then we will have to check all floors from **1-13** with the second ball and it will require a maximum of 13 drops to check all 13 floors . So the total no of drops are **14** in worst case and these are the minimum drops required to find the critical floor in worst case.

* Now lets understand this with another example:
* If the first ball breaks from **60th floor** then we know the last safe floor was **50th** from above, then we will use the second ball to check from **51 to 59** until the ball breaks. Suppose it breaks on **56** then the highest safe floor is **55.** So the total no of drops for the first ball will be 5 (As it's dropped from floor **14 27 39 50 60**) and for the second ball will be **6** (as it will be dropped from **51 52 53 54 55 56** in this case). So total drops are **6+5 = 11** that is less than **14** (worst case and maximum no of drops

CHAT GPT VERSION

* Start with the total number of floors = 100.
* We have two clay balls to test with
* Calculate the initial floor to drop the first ball:
* Find the smallest number x such that the sum of numbers from 1 to x is at least 100.
* The formula to find x is

* Solve this to get x ≈ 14.

Set the first drop floor to x = 14.

Set the next drop floor increment to x-1 = 13

Repeat the following steps until the first ball breaks or you reach the top floor:

* **Step 1:** Drop the first ball from the current floor.
* **Step 2:** If the ball breaks:
* Go to **Step 7**.
* **Step 3:** If the ball does not break:
  + Move to the next floor by adding the current increment to the floor number.
  + Decrease the increment by 1.
* **STEP 7 :If the first ball breaks:**
* Use the second ball to check all floors below the last safe floor (the floor just below where the first ball broke).
* Start from the last safe floor and go up one floor at a time until the ball breaks.
* Output the highest floor from which the ball can be dropped without breaking.
* Output the total number of drops required.

QUESTION NO : 1.5

(BY ROLL NO 24030)

SOLUTION :

Total amount to make minimum number of bags is **RS :277**

* Start with a bag containing RS : 1
* The next bag should contain RS : 2 (double the previous amount)
* The next bag should contain RS : 4 (double the previous amount) and so on
* Continue until the sum of bags reaches RS : 277
* Make 8 bags of RS 1, 2, 4 , 8, 16 , 32 , 64 , 128
* Sum of amount in these 8 bags is RS 255
* 9th bag will contain the amount (RS 277 -sum of amount in above 8 bags ) which is RS 22
* Final bags are :Bag 1: RS: 1

Bag 2: RS: 2

Bag 3: RS: 4

Bag 4: RS: 8

Bag 5: RS: 16

Bag 6: RS: 32

Bag 7: RS: 64

Bag 8: RS: 128

Bag 9: RS: 22

Let's take few examples

EXAMPLE: 1 FOR RS : 197

1. First take bag which contain closest amount from RS : 197 which is Bag 8
2. Then will take Bag 7
3. With these 2 bags the total amount becomes RS : 192
4. By giving Bag 1 and Bag 4 we can give RS 197
5. Finally minimum number of bags to make amount **RS : 197** is 4

EXAMPLE : 2 FOR RS: 41

1. First take bag which contain closest amount from RS : 41 which is Bag 6
2. Then will take Bag 4 and Bag 1
3. Finally by using 3 bags we are able to give total amount which is **RS : 41**

ARGUMENTS:

* Using amounts that are powers of 2 (1, 2, 4, 8, etc.) allows you to efficiently cover every possible value from RS 1 to RS 255 with the fewest number of bags.
* This method minimizes the total number of bags needed, and adding a final bag with RS 22 ensures that all amounts from RS 1 to RS 277 can be formed,

CHAT GPT VERSION (WITH CHANGES)

* Set the total amount to RS : 277
* Initialize bag value = 1
* Create bags with values that are double than the value in the previous bag like (1, 2, 4, 8, 16, etc.)
* Continue doing that until adding another bag would exceed the total amount
* Calculate remaining amount = total amount - sum of amount in bag
* Create last bag with remaining amount
* Use these bags to give any amount

**QUESTION NO:1.6**

(BY ROLL NO 24018)

SOLUTION :

* We have two cubes, with **6** faces on each cube. Lets name them as **A** & **B**.
* On cube **A**, label **3** **sides** with the numbers 0, 1, & 2.
* Mark the remaining three sides with 3, 4, & 5.
* On cube B, label 3 faces with 0, 1, & 2 as well.
* The remaining 3 faces should be marked with 6, 7, & 8.
* 9 doesn't need to be included separately, as 6 can be used instead, by flipping it upside down.
* Note:Number 7 should be represented as 07 instead of 7.

PSEUDO CODE BY CHAT CPT :

* Label 3 faces of cube A with numbers 0, 1, and 2 4.
* Label the remaining 3 faces of cube A with numbers 3, 4, and 5.
* Label 3 faces of cube B with numbers 0, 1, and 2.
* Label the remaining 3 faces of cube B with numbers 6, 7 , and 8.
* Note: Number 9 is not included separately, as 6 can be flipped upside down to represent 9.

QUESTION NO : 1.7

(BY ROLL NO 24030)

SOLUTION :

* We have **75** white beans and **150** black beans in a pot and a pile of black beans
* Remove two beans at a time
* If two white beans are removed, they are discarded, and one black bean is added to the pot.
* If a pair contains at least one black bean , remove that black bean and put the other bean in pot regardless of its color
* white beans are only removed in pairs
* As we know that initially we have 75 white beans which are in odd so; one white bean will always remain in pot
* Continue doing random selection until one last bean is left which is **white**

ARGUMENT:

Since we started with odd number of white beans and white beans are always removed in pairs so at the end 1 white bean will left

CHAT GPT VERSION ( WITH CHANGES )

* Initialize white beans = 75
* Initialize black beans = 150
* **Step 1:** Remove two beans from the pot.
* **Step 2:** Check the colors of the two beans:
* If both beans are white than discard both of them and add a black bean
* If one of them is black discard it and put the other bean white or black in pot
* Output = 1 white bean left in pot

BY ROLL NO 24018 :

<https://chatgpt.com/share/c99254df-7c1a-4392-a46e-93c00f467dfd>

BY ROLL NO 24030:

<https://chatgpt.com/share/584f9d9f-1b59-4674-9afb-275933db48e3>

QUESTION:1.9

BY ROLL NO 24030

SOLUTION:

The strategy to make infinite regions with infinite number of lines is as following

* No two lines should be parallel to each other
* Every line should intersect with every other line at different points
* Each intersection should divide an existing region into two
* No two lines should have a common intersection

Let us take a few examples to understand this strategy

* With **0 lines** there is only 1 plane which is **infinite** Infinite plane
* With **1 line** u get **2 regions**



* With **2 lines** (if they intersect), you get **4 regions.**

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* With **3 lines**, if each new line intersects all the previous lines at different points, u get **7 regions**

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* **IN GENERAL :**

| **N0 OF LINES (n)** | **NO OF NEW REGIONS** | **TOTAL REGIONS** |
| --- | --- | --- |
| 0 |  | 1 |
| 1 | +1 | 2 |
| 2 | +2 | 4 |
| — | — | — |
| — | — | — |
| n-1 | +( n-2) | (n-2)+(n-1) |
| n | + (n-1) | (n-1)+n |

* This pattern emerges because each new line can intersect all the previous lines, and each intersection adds new regions
* The formula for maximum number of regions **R(n)** formed by **n** lines is:
* The reason to add 1 in the formula is because with **0 lines** we still have **1 region** which is necessary to be included for accurate answers

PSEUDO CODE

STEP 1: Start with **n** number of lines

STEP 2: create a variable to store the number of regions

* Start with **1** because with **0** lines we have **1** region
* Put **regions = 1**

STEP 3: for each line from **1** to **n**

* Add the current line number to regions
* Continue doing this until u reach **n lines**
* By adding new line , more regions can create by intersecting the new line with the previous ones

STEP 4 : The number of regions can also be calculated by using the formula :

* Reason to add 1 is because when there are no lines on the plane (n=0 ), the entire plane is just one infinite region.

STEP 5 : Output the total regions

QUESTION NO : 1.10

BY ROLL NO 24030

SOLUTION:

* Initially duck is at the center of the pond
* If it simply swims to the edge of the pond covering a distance **D=r** the fox can easily catch him because the fox is **4 times faster** the speed of duck
* Let's make a strategy for the duck
* The duck should start moving in a small circle around the center of the pond. By doing so, the duck can make the fox run along the edge of the pond, which is a much larger circumference compared to the duck's small circle.
* Lets imagine in duck swims at **r/4** of radius of pond in this case :
* Distance covered by fox = **2𝝅r**
* Speed of fox = 4 times speed of duck = **4Vd**
* Distance covered by duck = **2𝝅(r/4)**
* Speed of duck = **Vd**
* If we calculate the time taken to cover the distance for both of them, we find that they complete their circles in exactly the same time. In this scenario, the fox has the advantage.

Time taken by fox : time taken by duck

2𝝅r/4Vd : 2𝝅(r/4) / Vd let : r= 1

2𝝅/4Vd : 2𝝅/4Vd

CASE : 1

* Now if we take a smaller distance than **r/4** will it help the duck to escape?
* When the duck is swimming in a circle whose radius is less than **r/4** it completes the circle faster than the fox; it covers a shorter distance in each lap compared to the fox, despite the fox being faster.
* If it continues swimming in that smaller circle after each turn the fox will gradually lose its advantage .eventually the time will come when they are exactly opposite to each other
* When this happen the duck can swim covering distance **D ≅ 3r/4** to the edge opposite to the fox.the duck will reach the land before the fox and successfully escape

CASE : 2

* If the duck swims in a circle with a radius greater than **r/4** , the increased distance it must cover allows the fox to keep up or even gain on the duck, due to the fox's superior speed.
* In this scenario, the fox maintains its advantage, making it more likely to catch the duck before it fly from the land
* Therefore, swimming in a larger circle diminishes the duck's chances of

escaping.

PSEUDO CODE (WITH CHANGES)

**1. Start:**

* Duck is at the center of the pond **(Point A).**
* Fox is at the edge of the pond **(Point B).**
* Fox is **4 times** faster than the duck.

**2. Basic Idea:**

* If the duck swims directly to the edge**:** The fox will catch the duck because it is faster.
* **The duck should swim in a small circle** around the center of the pond

**3. Strategy:** CASE : 1

* **Step 1:** Duck begins swimming in a small circle with a radius less than one-fourth of the pond's radius.
* **Step 2:** As the duck swims in this small circle, the fox will have to run along the edge of the pond, covering a much larger distance.
* **Step 3:** Since the duck covers a shorter distance, it can complete its circles faster than the fox.
* **Step 4:** The duck continues to swim in circles until it reaches a position where the fox is directly opposite it on the other side of the pond.

**4. Escape Plan:**

* **Step 5:**The duck should then swim directly to the edge of the pond (covering a distance of about **3r/4**) opposite to the fox
* **Step 6:** The duck reaches the land before the fox and safely escapes.

**6. Outcome:**

* **If the duck follows the strategy correctly:**
  + The duck escapes successfully.
* **If the duck swims in a larger circle:**
  + The fox will catch the duck.

CASE 2:

If the duck swims in a circle with a radius larger than r/4 :

* The larger distance makes it harder for the duck to stay ahead.

**7. OUTPUT :**

* The fox will maintain or increase its advantage and is likely to catch the duck before it can reach the land.

QUESTION:1.11

BY ROLL NO: 24018  
Solution:  
FOR 1.5 HOURS  
-Burn the first string from one side.   
-When burned completely, 1 hour has passed.   
-Then, burn the second string from both sides. It will take 30 mins to burn fully.   
Note: Don't burn both strings at once.   
  
PSEUDOCODE   
-Burn the first string from one side.  
-Wait until the first string is fully burned (this takes 1 hour)  
-Burn the second string from both sides simultaneously.  
-Wait until the second string is fully burned (this takes 30 minutes)  
-The total time taken is 1.5 hours (1 hour + 30 minutes)  
  
FOR 45 MINS  
-Burn string A from one side.   
-Burn string B from both sides.  
-In 30 mins, string B is completely burned while string A still has to burn for 30 mins.   
- Now, burn string A from the other side as well(it is now burning from both sides)

-It will now burn in 15 mins only.   
-Total time taken is 45 mins.

PSEUDOCODE:  
-Ignite string A at one end.   
-Simultaneously, ignite string B at both ends.   
-Wait 30 minutes until string B is fully burned.   
-After 30 minutes, ignite the other end of string A.  
-Wait 15 minutes until string A is fully burned.  
-Total time elapsed is 45 minutes.  
  
 QUESTION NO 1.12  
 BY ROLL NO : 24018

SOLUTION:  
-Take number 1 & 2 across the river. We will be charged 2 for this.   
-Bring number 1 back while leaving number 2 there. We will pay 1 for this.   
-Now take number 5 & 10 to the other side. We will be billed 10 for this.  
-Bring number 2 back. We will pay 2 for this.   
-Atlast, take numbers 1 & 2 to the opposite bank. We will pay 2 for this.   
-Total minimum cost to take all the numbers across the river will be 17.   
  
PSEUDOCODE   
-Take numbers 1 and 2 across the river (cost = 2).  
-Return with number 1 (cost = 1).   
-Take numbers 5 and 10 across the river (cost = 10).   
-Return with number 2 (cost = 2).   
-Take numbers 1 and 2 across the river again (cost = 2). Total cost = 17.

**QUESTION 1.13**

**BY ROLL NO : 24018**SOLUTION:  
-Consider the number of people to be X. (Let's suppose they are 10^15)  
-Lets increase exponentially, in multiples of 10.   
-Give the first antidote to 10^0 and the next one to 10, third one to 10^2, 10^3 and so on.   
-In only 15 antidotes, we will figure out the range of people affected by the virus.   
-Now, let's consider the range from 10^14 to 10^15.   
-We will use a halving technique.   
-Number of antidotes required for 10^14 to 10^15 range will be calculated as follows:  
 log2(9x10^14)= 45 antidotes   
-Previous antidotes required while increasing exponentially=15   
-Total number of antidotes= 15+45= 60  
  
PSEUDOCODE  
-Set X to 10^15.

-Distribute antidotes exponentially:

* Administer antidotes at 10^0, 10^1, 10^2, ..., up to 10^14 (15 antidotes).

-Identify the affected range between 10^14 and 10^15.

-Use halving technique:

* Calculate additional antidotes as log2(9x10^14) = 45.

-Total antidotes required = 15 + 45 = 60.

OPEN CHALLENGE

BY ROLL NO 24030

SOLUTION:

1.INITIAL PAIRING:

* Pair up all **100** tribesmen into **50** pairs
* For each pair **(A,B)** ask **A** if **B** is trustworthy and ask **B** if **A** is trustworthy

2.EVALUATE RESPONSES:

* **Case 1**: **A** says **B** is trustworthy, and **B** says **A** is trustworthy. Both are likely trustworthy.
* **Case 2**: **A** says **B** is a faker, and **B** says **A** is a faker. At least one is a faker.
* **Case 3**: **A** says **B** is trustworthy, and **B** says **A** is a faker (or vice versa). One is likely a faker.

4.ELIMINATION:

**Reject Conflicting Pairs**:

* For **Case 1** pairs, keep one person **(say A)** for the next round and discard B.
* For **Case 2** and **Case 3** pairs, discard both individuals. They are not trustworthy.

**Halve the Group**: You should now have approximately half the original group size remaining.

5. REPEAT THE PROCESS :

* Continue pairing and questioning the remaining tribesmen
* Each round halves the group size further and you continue until u have a very small group

6.HANDLE ODD NUMBERS:

* If you have an odd number of people in any round, set one aside and proceed. You can test the isolated person in the next round.

**7.FINAL IDENTIFICATION :**

* As the group size shrinks, trustworthy individuals become more apparent.
* Eventually, with fewer than 4 people, you can directly ask and identify the trustworthy ones.

8.WORST CASE SCENARIOS:

* Total Questions: The worst-case scenario involves asking questions in a geometric series: 100 + 50 + 25 + 13 + 7 + 3 = 198 questions. Even if you have to handle odd numbers, the total questions asked will be close to, but not exceed, 300.
* Trustworthy Majority: At each stage, you’re preserving the majority of trustworthy people, ensuring that at least one trustworthy person is identified.

PSEUDO CODE

* Pair up the tribesmen and ask if they trust each other.
* Evaluate their responses and decide who to keep or discard.
* Halve the group and repeat the process until the group is very small.
* Handle odd numbers by setting one person aside if needed.
* Directly identify the trustworthy tribesmen when the group is small enough.
* Worst-case scenario: Expect to ask up to 300 questions, but you'll find trustworthy individuals.

ROLL NO: 24018

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ROLL NO : 24030

<https://chatgpt.com/share/5ff2a34e-83ee-4555-8d0c-f308ccf80eb4>